Value Weighting and Simple Optimization Of Portfolios: An Empirical Examination

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Abstract

Strategy might benefit from theory. Then again, tests of strategy do not always confirm theory. This paper examines whether individual investors who invest in a small number of stocks can achieve returns superior to the value-weighting approach of portfolio theory by employing a simple optimization scheme. The motivation for the study rests on an appreciation of the theory coupled with the recognition of the possible violation of assumptions integral to the theory. Markowitz (1952, 1959) fathered and revolutionized modern portfolio management by demonstrating how to achieve efficient diversification. Markowitz's assumptions and theory prescribe that the optimal way for all investors to allocate capital at risk is to invest in the "market portfolio," a value-weighted portfolio that includes all risky assets. The popularity of index funds designed to approximate a market portfolio is evidence of the profound influence of portfolio theory on portfolio management. Although the theory never alludes to *ex-post* performance, real world considerations make it unlikely that portfolio behavior will strictly follow the predictions of portfolio theory. "Imperfections" such as restrictions on short selling, transactions and management costs, and non-divisibility of assets make it unlikely that value-weighting will produce a portfolio that is *ex-ante* efficient. These deficiencies are especially relevant for individual investors who may spread their investment across only a few securities. The empirical results are interesting in terms of theory as well as practical applications.

Introduction

Markowitz (1952, 1959) forever changed portfolio management by characterizing and demonstrating how to achieve efficient diversification. Markowitz proved that, under certain simplifying assumptions (such as homogeneous expectations), all investors agree about the optimal way to allocate capital at risk. Since all investors select the same allocation for the portion of capital they place at risk, the theoretically optimal portfolio must be an all-inclusive value-weighted portfolio. The optimal portfolio is, by definition, the market portfolio.

Today, the popularity of index funds designed to mimic the market portfolio is evidence of the profound influence of portfolio theory on portfolio management. Inquiring minds-and practical people-want to know if value-weighting is the best way to put theory into practice.

Portfolio theory prescribes behavior in an ex-ante setting, but is silent with respect to the characterizations of ex-post outcomes. Real world considerations make it unlikely that a portfolio will behave exactly as theory predicts. "Imperfections" such as restrictions on short selling, heterogeneous expectations, transactions and management costs, and non-divisibility of assets make it unlikely that value-weighting will produce a portfolio that is ex-ante efficient. These considerations are especially relevant for individual investors who may spread their investment across only a few securities. Nevertheless, the elegance and compelling logic of the theory prompt attempts to apply the theory even though practitioners recognize the variance between the simplifying assumptions of the theory and the realities of the world.

This paper examines whether individual investors who allocate capital among only a small number of stocks can, by employing a simple optimization scheme, achieve returns that are superior to a value-weighting approach using the same small number of securities. The empirical results are interesting in terms of theory as well as practical application.

Background

The seminal work of Markowitz spawned profound implications for decisions on how to expose capital at risk. The simplicity of the conclusions of his monumental work adds to the elegance of his work and theory. For those who desire to expose capital to risk in hopes of return, the prescription is simple. For each person, all capital at risk should find a home in "the market portfolio." The individual attains the desired level of risk by investing the correct proportion at risk in the market portfolio, with the remainder borrowed or lent at the riskless rate.

In this scheme, all who desire risk should order the same entrée-just in different-sized portions. The precipitate of this notion implies that one and all of those who desire to have capital at risk collectively both define and share ownership of the same portfolio-the market portfolio.

Roll (1977) focused attention on the impossibility of actually constructing and/or using the "market portfolio" in practice, as well as in empirical examinations. Other clever researchers have subjected the canons of portfolio theory to a barrage of empirical scrutiny. A minute sampling of the many excellent contributions includes Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), Gibbons (1982), Fama and French (1993), and Daniel and Titman (1997).

Practitioners have endorsed the concepts and implications of portfolio theory and employed the dictates of the theory to guide the investment of hundreds of billions of dollars. Numerous researchers, such as Statman (1987), also have recognized and demonstrated that even with a random selection of stocks, employing a relatively small number of assets achieves high levels of diversification. In practice (at least with respect to financial assets), investors apparently can approximate the perfectly diversified portfolio, even if perfect diversification is unattainable.

Various proxies for the market portfolio underlie an abundance of empirical work and serve as (admittedly deficient) attempts by investment professionals to attain the degree of diversification that the true market portfolio provides. We recognize the deficiencies pointed out by Roll. At the same time, we observe the commitment of massive amounts of capital governed by the canons of the theory and independent of the shortcomings illuminated by Roll.

Our examination pits a simple optimization scheme that selects a small number of stocks for portfolios against the results derived from the value-weighting approach of portfolio theory. We need not enter the debate on the index. This empirical work does not rely in any meaningful way depend on a proxy of the market portfolio. We include CRSP's value-weighted index merely as a benchmark measure of return for a very well diversified market portfolio.

However, our effort does suffer from a conceptual deficiency too often overlooked by researchers, that portfolio theory rests in an expectations context. The theory does not characterize the behavior of returns relative to risk that should surface in an ex-post analysis.

Methodology

Biased sample

The choice of sample hinged on the desire to select popular stocks that individual investors often include when forming portfolios. The selection of the relatively small number (nine) reflects the modest number of stocks many individuals hold in their portfolios. Although the sample is biased toward large stocks, the sample is relevant given the behavior of investors and serves us well in making the desired comparison.

Data

The Center for Research in Security Prices (CRSP) provided monthly data for nine randomly selected, widely followed stocks that individuals might be interested in selecting as investments. The selection is random given we picked from a sub-universe of stocks of relatively broad holding. Table 1 identifies the stocks chosen for the study.

Period of Study

The examination employed data from December 1992 through December 1997. We identified each stock's market capitalization and monthly returns, assuming reinvestment of dividends. We also collected contemporaneous monthly returns for the value-weighted portfolio that includes all stocks contained in the CRSP database. Represented by "VWI", the performance of this index allows us to compare the performance of our narrowly diversified portfolios to that of a highly diversified portfolio.

The effort requires monthly data prior to the date of portfolio formation, in this case a thirty-month rolling period. Hence, the actual formation of portfolios begins with June, 1995. As we proceeded to successive months, the thirty-month period gained the most recent prior month's data and dropped the earliest month's data.

At the beginning of each month, starting with June 1, 1995, we formed two portfolios:

* **A value-weighted portfolio**, based on the market values of each company's equity on that date and
* **An optimized portfolio**. The formation of the optimized portfolios required calculating average returns, standard deviations, and all cross-correlations for the 30-month period that immediately precedes the portfolio formation date. The resulting data array served as the input for the optimization program (described below) contained in Shimko, Foster, and Will (1998).

We constrained optimized portfolios to contain no short positions to more closely approximate actual conditions faced by individual investors. The holding period for each portfolio was one month. At the beginning of each successive month, we rebalanced the value-weighted portfolio and created a new optimized portfolio based on the most recent 30 months' data. This process continued through December 1, 1997, providing a monthly return series for each portfolio that contains 31 observations. Although continuous rebalancing to maintain an optimized portfolio is both desirable and conceptually possible, such frequent adjustments would impose unacceptable transaction costs. We chose a monthly rebalancing frequency as a compromise between these two opposing goals.

The Optimizer

Investors employ suggestions, models, schemes, and advice from many sources. Shimko, Foster, and Will (1998) is one such source that provides several quantitative tools, including an optimization algorithm. Investment advisors (and common sense) generally assert that an investor should assess personal circumstance and decide on the appropriate level of risk exposure for the person. Indeed, this notion of deciding on appropriate and desired risk before allocating capital is a core conclusion of portfolio theory. This perspective is a key factor that influenced our decision to employ this particular optimizer.

The optimizer included in Shimko, Foster, and Will (1998) requires the user to input her coefficient of risk aversion and assumes the following expression characterizes her utility function:

$$U_i = \bar{r}_i - 0.005 A \sigma_i^2$$

where,

U_i = the utility provided by security i,
$\bar{r}_i$ = the average return for security i,
A = coefficient of risk aversion, and
σ_i^2 = the variance of return for security i,

The software's algorithm determines an efficient portfolio given an investor's specified coefficient of risk aversion. Recall that the minimum variance set consists of those portfolios that minimize risk for a given level of return. Figure 1 demonstrates that the minimum variance set is bullet shaped - its shape determined by the risk-return opportunities available to investors at a particular point in time. Moreover, portfolios on the upper half of the minimum variance set dominate any other portfolios since they offer greater expected returns without an increase in attendant risk. These dominating portfolios make up the efficient frontier. The min-max criterion directs investors to the efficient frontier, opportunities on the northwest quadrant of the minimum variance set that offer superior expected returns for any given level of risk.

For larger risk aversion coefficients, the optimizer selects a portfolio closer to the tip of the bullet (the global minimum variance portfolio). Investors with smaller risk aversion coefficients are directed to efficient portfolios that are farther up the frontier, away from the tip of the efficient frontier bullet.

To explore potential sensitivity of results to risk aversion, for each portfolio formation date we calculated optimized portfolios for investors with coefficients of risk aversion of A = 2, 4, 6, and 8. In the following section, we describe how the optimizer works and present our results.

Results

Table 2 offers a summary of the results. The first column of Table 2 contains the portfolio formation dates. The second, third, fourth, and fifth columns contain monthly returns for optimized portfolios for coefficients of risk aversion equal to 2, 4, 6, and 8 respectively. The sixth column (headed **VW**) reports the return on the value-weighted portfolio containing the nine stocks included in our sample. The last column (headed **VWI**) is the return on the CRSP value-weighted portfolio composed of every stock contained in the CRSP database. The bottom of the exhibit reports the maximum, minimum, average, and standard deviation for each return series.

The optimizer selects allocations based on historical data and accounts for the investor's degree of risk aversion. Using the ex-post data, the optimizer selects an efficient allocation for the investor. Given a range of risk aversion coefficients, the optimizer will identify a corresponding set of efficient portfolios. The investor with the lowest risk aversion coefficient receives the portfolio with the highest return and the investor with the highest risk aversion coefficient receives the portfolio with the lowest return. Investors using the optimizer to allocate capital would hope that the chosen portfolio's ranking would persist.

To illustrate, we identify four efficient portfolios for each month. Of these, the optimizer assigns the portfolio with the highest ex-post return to the investor with the lowest risk aversion (which is 2 in this paper). This relatively risk-tolerant investor hopes that the portfolio with high ex-post returns will produce high returns during the next month.

Similarly, an investor with risk aversion coefficient of 8 (highest tested risk aversion) receives the portfolio with the lowest ex-post return. This investor also hopes that the ex-post characteristics of the portfolio will persist into the future and is willing to accept lower expected return in exchange for less risk exposure.

Our results suggest the following conclusions.

- **Optimization based on ex-post data does not yield predictable ex-ante return outcomes.**

  - **Focusing only on the maximum returns and minimum returns for optimized portfolios suggests that ex-post characteristics of optimized portfolios do persist.** With one exception (A = 6), the maximum return for optimized portfolios decreases as risk aversion increases. In every case, the minimum return for optimized portfolios decreases as risk aversion increases.

  - **Average returns, however, exhibit exactly the opposite behavior, increasing as risk aversion increases!** The average return for optimized portfolios ranges from 1.64% (for A = 2) to 2.49% (for A = 8). Strangely, optimization provides the most risk averse investors the greatest average return, and the most risk tolerant investors earn the lowest average return. Our data suggests that simple optimization does not produce portfolios with time-consistent performance rankings with respect to average return. In fact, the results are exactly the opposite of investors' ex-ante preferences. We will offer a potential explanation for this puzzling result below.

- **Optimization based on ex-post data does not yield predictable ex-ante risk outcomes. Of course, return is only half of the story for investors.** Optimized portfolios based on ex-post data will select allocations with lower ex-post standard deviation for investors with higher risk aversion, and vice-versa (think of moving from right to left on the graph of the efficient frontier).

- **The optimizer apparently does not generate portfolios that retain their ranking with respect to standard deviation.** The different values of A resulted in the following standard deviation of returns:

These results indicate the investor with the highest degree of risk aversion winds up with the lowest standard deviation and, the investor with the highest degree of risk tolerance achieves the highest standard deviation. However, no obvious inverse relationship exists between risk aversion and standard deviation.

Risk Aversion Coefficient	Standard Deviation of Return
2	5.61%
4	5.37%
6	5.58%
8	5.31%

- **Optimizations based on ex-post data do yield predictable reward-to-risk outcomes.** The final row of the table reports the ratio of average return to standard deviation for each of the optimized portfolio return series. A clear pattern emerges for the ratio in spite of irregularities in its components. Consistent with the prescriptions of theory, investors with the lowest degree of risk aversion earn the lowest average return per unit of risk, and the reward-to-risk ratio sharply increases as risk aversion increases. These results are consistent with the notion that risky opportunities in expected risk-return space result in an efficient frontier of changing slope with the result that increasing risk exposure increases expected return, but at a decreasing rate. The lower an investor's risk aversion coefficient, the farther her risky portfolio from the minimum risk (eastern most) point of the bullet.

- **When investing in a small number of stocks, value weighting beats simple optimization.** The next-to-last column (VW) of Table 2 reports monthly returns for a value-weighted portfolio of our nine sample stocks. The results are compelling:

  - The value-weighted portfolio's maximum return is less than the maximum return of any of the optimized portfolios we examined.

  - Similarly, the value-weighted portfolio's minimum return is greater than the minimum return of any of the optimized portfolios.

- The narrower range for the value-weighted portfolio indicates that value weighting produces a portfolio that is less volatile than an optimized portfolio. The value-weighted portfolio's standard deviation (4.34%) confirms this: it is almost a full percentage point less than the smallest standard deviation for the optimized portfolios. Moreover, the value-weighted portfolio posted a higher average return over the sample period than any optimized portfolio.

- For the value-weighted portfolio, average return is 58.36% of standard deviation. The highest average return-to-standard deviation ratio for the optimized portfolios is only 46.85%. Figure 2 depicts average monthly return and standard deviation of return for each of the optimized portfolios (2, 4, 6, and 8), the nine-stock value-weighted portfolio (VW), and the CRSP value-weighted portfolio (VWI).

Including the CRSP value-weighted index allows comparisons between the strategies studied in this paper and that of investing in a widely diversified index. The comparisons yield interesting results. Value-weighting our nine sample stocks provided superior returns to any optimized portfolio, but achieving even greater diversification with the value-weighted index dominates the small value-weighted portfolio. Refer again to Table 2. Although the index's maximum return is less than the maximum for the nine-stock portfolio (7.64% for VWI, 10.68% for VW), the index's minimum return is larger than the smaller portfolio's minimum (-5.33% vs. -8.01%). The index's narrower range explains why its standard deviation is a full percentage point lower than the smaller portfolio's (3.34% vs. 4.34%). Even though the index's average return is smaller than the smaller portfolio (2.08% vs. 2.53%), the index portfolio enjoys the highest reward-to-risk ratio of any portfolio studied.

The results emphasize that small investors should never underestimate the importance of diversification, even imperfect and relatively simple diversification. Although the optimization technique used here exactly identifies ex-post efficient portfolios, the technique apparently suffers deficiencies. Optimization prescribes a poor strategy for portfolio formation since optimized portfolios almost always end up being poorly diversified. The ex-post optimal allocation does not diversify away unique risk, which results in the unfavorable outcome shown in Figure 2 for the optimizations. A method that held out possible offers of advantage over simple value-weighting could not overcome the abandonment of diversification even when dealing with only nine securities - or more aptly, because we are dealing with only nine securities.

Table 3 details investment weights for optimized portfolios and the value-weighted portfolio for July 1, 1996. These results are typical. With low risk aversion, the optimizer prescribes "plunging" in just one or two assets. Very often, investors place the greatest majority of funds into just one stock, as is the case here. However, diversification increases as risk aversion increases. Even when the number of stocks included in the portfolio does not increase with risk aversion, the maximum amount allocated to any one stock decreases.

We demonstrate that the investment strategy that relies on applying an optimization technique to a small number of stocks ignores Markowitz's most important result, to the detriment of the investor. For the nine stocks we considered in this study, value-weighting beats the optimization technique because value-weighting guarantees that the portfolio will include at least some of every stock under consideration. This line of reasoning also explains why the performance of VWI is so dominating. The CRSP index portfolio contains thousands of stocks, providing a very high degree of diversification.

Observations and Summary

Observations

Our attention returns to certain aspects of the findings. First, even when investing in a small number of stocks, our results show it is unwise to abandon the mandate of portfolio theory. Aside from another drop of evidence in support of portfolio theory, practical realities add relevance to our results. The value-weighting approach endorsed by our findings is relatively easy to implement. First, periodic rebalancing to adjust for price changes is easier and less expensive than ever before, thanks to increased competition and on-line trading. Second, since individuals' portfolios typically contain only a few stocks, calculating portfolio weights is not difficult. Finally, the elaborate calculations required to use optimization are neither necessary nor beneficial.

Others have ably demonstrated mathematically how low correlations between a small number of component stocks reduce variance of a portfolio's expected return. We entertain speculations on another contributing reason why so few stocks seem to get the job done. Price behavior stems from human behavior rather than a need to conform to the logic of theory. Relatively simple conscious algorithms may both characterize and offer benefit to investors. Indeed, researchers have reported on their investigations in this area.[1] Behavior that employs relatively simple and acceptable conscious approaches to complex situations may influence the pricing of securities in a manner that differs from the prescription of theory. The result is a precipitation of observed ex-post price/return/risk relationships that represent the results of a relatively simple approach- with good effect - to a theoretical optimal challenge.

Some of our results are counterintuitive and suggest topics for future research. For example, optimized portfolios for the most risk averse investor earned the greatest average return over the sample period, while the least risk averse investor's optimized portfolio earned the lowest average return. Jegadeesh (1990) and Jegadeesh and Titman (1993) have documented reversals in stock returns that might explain our results. In each of these papers, the researchers find evidence that security returns measured over adjacent time intervals exhibit negative serial correlation and hypothesize that their results are caused by the tendency of traders to overreact to news. If their hypothesis is correct, stocks that enjoy unusually good performance during the periods that immediately precede our portfolio formation dates may simply be beginning to revert to their long-term mean return. Assigning a portfolio with high ex-post returns to an investor with low risk aversion would cause the investor to, on average, earn lower returns after the portfolio formation date. Why reversals occur and whether they can be expected to persist are questions that require further research to be answered.

Summary

This study compares a portfolio optimization technique to a value-weighted approach to investing. The results suggest that relatively simple optimization schemes provide inferior risk-adjusted ex-post returns. We attribute the inferior returns of optimized portfolios to a weighting effect. The tendency of heavily weighting very few stocks in optimized portfolios causes a higher standard deviation compared to value-weighted portfolios that include all available stocks.

Although alternate schemes surface and proponents offer these alternatives as potential improvements over relatively simple methods (such as value-weighting), our results lead us to join the chorus of praises of Markowitz's work. Even absent the simplifying perfect market assumptions that support the theory, the common sense approach that portfolio theory prescribes is strong medicine. A simple value-weighted portfolio of relatively few securities offers advantage over the optimization technique we studied. Second, the portfolio approach likely inoculates the individual investor against the pitfalls of more elaborate approaches to investing.

Table 1
Stocks In The Sample

Stock	Symbol
Boeing	BA
DuPont	DD
Eastman Kodak	EK
Ford	F
General Electric	GE
Goodyear Tire and Rubber	GT
International Business Machines	IBM
Coca-Cola	KO
AT&T	T

Figure 1
The Minimum Variance Set and the Efficient Frontier

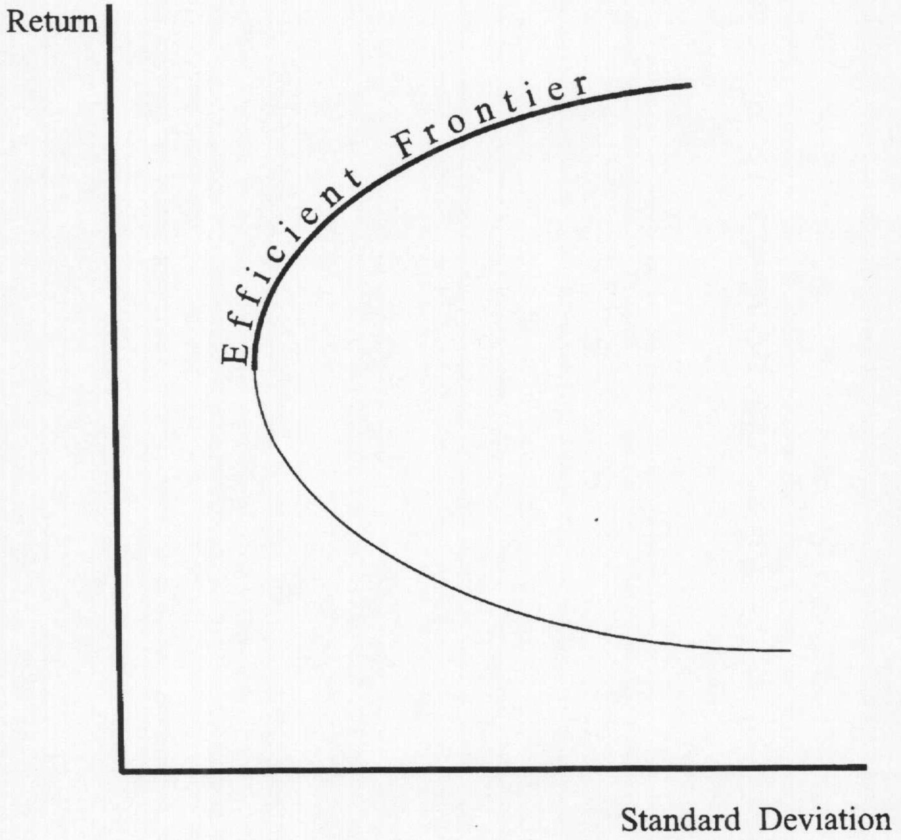

Table 2

Portfolio Return

Date	Returns on Optimized Portfolios for Various Levels of Risk Aversion				Value Weighted Return	Return on Value Weighted Index
mm/dd/yy	A = 2	A = 4	A = 6	A = 8	VW	VWI
6/1/95	2.74%	3.22%	3.29%	3.29%	2.11%	3.09%
7/1/95	3.99%	3.82%	3.44%	3.17%	3.08%	4.07%
8/1/95	-4.59%	-4.47%	-4.12%	-3.62%	0.24%	0.95%
9/1/95	0.05%	1.00%	1.92%	2.51%	6.57%	3.64%
10/1/95	-2.17%	-1.73%	-0.61%	0.45%	-0.95%	-1.09%
11/1/95	2.08%	3.62%	4.77%	5.35%	4.67%	4.30%
12/1/95	0.14%	-0.55%	-0.80%	-0.92%	1.48%	1.55%
1/1/96	0.77%	3.70%	4.49%	4.65%	5.96%	2.82%
2/1/96	11.98%	9.93%	8.26%	7.80%	2.00%	1.60%
3/1/96	-8.36%	-3.46%	-1.73%	-0.86%	1.29%	1.16%
4/1/96	-3.78%	-4.14%	-3.89%	-3.58%	-0.63%	2.54%
5/1/96	1.69%	2.12%	4.08%	5.03%	4.65%	2.69%
6/1/96	4.20%	3.51%	3.28%	3.21%	1.23%	-0.79%
7/1/96	-4.05%	-3.16%	-2.78%	-2.60%	-4.07%	-5.33%
8/1/96	7.20%	6.85%	6.57%	6.03%	3.15%	3.23%
9/1/96	2.58%	2.91%	3.02%	3.20%	4.55%	5.30%
10/1/96	-0.20%	0.01%	0.09%	0.17%	1.29%	1.42%
11/1/96	5.36%	5.92%	5.75%	5.37%	7.50%	6.57%
12/1/96	0.54%	1.05%	1.22%	1.46%	1.86%	-1.14%
1/1/97	7.95%	8.38%	7.93%	7.61%	5.29%	5.31%
2/1/97	1.74%	1.80%	14.62%	12.94%	-0.01%	-0.08%
3/1/97	-8.40%	-8.28%	-7.95%	-7.61%	-5.85%	-4.45%
4/1/97	7.80%	8.83%	9.46%	9.78%	9.36%	4.26%
5/1/97	7.69%	7.77%	7.79%	7.79%	7.46%	7.13%
6/1/97	4.35%	3.93%	3.66%	3.38%	3.61%	4.42%
7/1/97	5.88%	6.12%	6.10%	6.34%	6.74%	7.64%
8/1/97	-7.87%	-9.30%	-9.81%	-10.07%	-8.01%	-3.63%
9/1/97	3.37%	3.34%	3.33%	3.32%	6.22%	5.80%
10/1/97	-5.66%	-5.51%	-5.70%	-6.02%	-4.95%	-3.41%
11/1/97	14.31%	14.09%	12.14%	10.58%	10.68%	3.11%
12/1/97	-0.53%	-0.75%	-0.83%	-0.99%	2.05%	1.84%
Max	**14.31%**	**14.09%**	**14.62%**	**12.94%**	**10.68%**	**7.64%**
Min	**-8.40%**	**-9.30%**	**-9.81%**	**-10.07%**	**-8.01%**	**-5.33%**
Average	**1.64%**	**1.95%**	**2.48%**	**2.49%**	**2.53%**	**2.08%**
Std Dev	**5.61%**	**5.37%**	**5.58%**	**5.31%**	**4.34%**	**3.34%**
Avg/SD	**0.292342**	**0.363972**	**0.445421**	**0.468521**	**0.583618**	**0.6235964**

Figure 2

Return-to Standard Deviation Ratios for Optimized and Value-Weighted Portfolios and the Value-Weighted Index

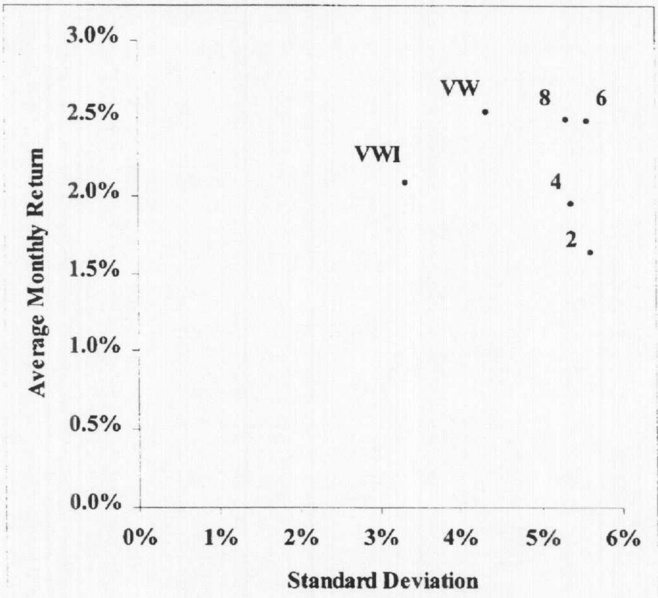

Table 3

Asset Allocations for July 1, 1996

Stock	Optimized Portfolios 2	4	6	8	VW
Boeing	4.88%	19.85%	18.79%	16.42%	5.34%
DuPont	0	0	1.82%	5.60%	7.86%
Eastman Kodak	0	0.87%	10.74%	14.61%	4.76%
Ford	0	0	0	0	6.33%
General Electric	0	0	0	0	25.55%
Goodyear	0	0	0	0	1.32%
IBM	0	0	2.31%	2.82%	9.54%
Coca-Cola	95.12%	79.28%	66.34%	60.55%	21.66%
AT&T	0	0	0	0	17.65%
Total	100.00%	100.00%	100.00%	100.00%	100.00%
Number of Stocks	2	3	5	5	9

Acknowledgment

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Endnote

1. Bower's (1999) "Simple Mind, Smart Choice" reports on some interesting notions in this area. However, these authors do not accept necessarily appropriate results as reported by Bower concerning the returns on portfolios examined.

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